Adaptation of the AHP as Multi Criteria Decision Making Approach and Testing the Original AHP over Two Evaluative Criteria

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Abstract — In a multiple criteria decision making (MCDM) problem, a decision maker (DM) often needs to select or rank alternatives associated with some usually conflicting attributes or objectives. These problems arise in many real-world situations. The final decision is based on the evaluation of a number of alternatives in terms of a number of criteria in many industrial and engineering applications. This Problem may become a very difficult one when the criteria are expressed in different units or the pertinent data are difficult to be quantified. The Analytic Hierarchy Process (AHP) is an effective approach in dealing with this kind of decision problems. This research paper is based on understanding working of the AHP technique in Multi-Criteria Decision-Making problems and examines AHP with respect to defined evaluative criteria.

Index Terms — Analytic Hierarchy Process, Multi Criteria Decision Making, Pair wise Comparisons, Consistency ratio.

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1 INTRODUCTION

The Analytic Hierarchy Process is a requirement prioritization technique that permits the evaluation of multiple diverse criteria, all of which affect the final decision. It was developed by Saaty in the 1970s and it is used around the world in a wide variety of decision situations, in fields such as government, business, industry etc. The AHP has attracted the interest of many researchers mainly due to the nice mathematical properties of the method and the fact that the required input data are rather easy to obtain. The pertinent data are derived by using a set of pairwise comparisons. These comparisons are used to obtain the weights of importance of the decision criteria, and the relative performance measures of the alternatives in terms of each individual decision criterion.

Some of the industrial engineering applications of the AHP include its use in integrated manufacturing [6], in the evaluation of technology investment decisions [12], in flexible manufacturing systems [7], layout design [4], and also in other engineering problems [5].

As an illustrative application considers the case in which one wishes to upgrade the computer system of a computer integrated manufacturing (CIM) facility.

There are a number of different configurations available to choose from. The different systems are the alternatives. A decision should also consider issues such as: cost, performance characteristics (i.e. CPU speed, memory capacity, RAM, etc.), availability of software, maintenance, expendability, etc. These may be some of the decision criteria for this problem. In the above problem we are interested in determining the best alternative (i.e., computer system).

In some other situations, however, one may be interested in determining the relative importance of all the alternatives under consideration. For instance, if one is interested in funding a set of competing projects (which now are the alternatives), then the relative importance of these projects is required (so the budget can be distributed proportionally to their relative importance). Multi-criteria decision-making (MCDM) plays a critical role in many real life problems.

2 STRUCTURE OF THE DECISION PROBLEM UNDER CONSIDERATION

The structure of the decision problem considered in this paper consists of a number, say M, of alternatives and a number, say N, of decision criteria. Each alternative can be evaluated in terms of the decision criteria and the relative importance (or weight) of each criterion can be estimated as well.

Let a_{ij} (i=1,2,3,...,M, and N=1,2,3,...,N) denote the performance value of the i-th alternative (i.e., A_i) in terms of the j-th criterion (i.e., C_j). Also denote as W_j the weight of the criterion C_j. The core of the typical MCDM problem can be represented by the following decision matrix in Fig. 1.

For the given decision matrix, the decision problem considered in this study is how to determine which the best alternative is. A slightly different problem is to determine the relative significance of the M alternatives when they are examined in terms of the N decision criteria combined. In a simple MCDM situation, all the criteria are expressed in terms of the same unit. But, in many real life MCDM problems different criteria may be expressed in different dimensions. The multiple dimensions situation makes the MCDM problem to be a complex one and the AHP approach offers a great assistance in solving this type of problem situation.

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Alter-native	**Criterion**						
	C_1	C_2	C_3	_	_	_	C_N
	W_1	W_2	W_3	_	_	_	W_N
A_1	a_{11}	a_{12}	a_{13}	_	_	_	a_{1N}
A_2	a_{21}	a_{22}	a_{23}	_	_	_	a_{2N}
A_3	a_{31}	a_{32}	a_{33}	_	_	_	a_{3N}
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
A_M	a_{M1}	a_{M2}	a_{M3}	_	_	_	a_{MN}

Fig. 1: Decision Matrix representing Criteria and Alternatives.

3. THE ANALYTIC HIERARCHY PROCESS

The AHP is an approach that uses a multi-level hierarchical structure of objectives, criteria, sub criteria, and alternatives. Since the early days it became apparent that there are some problems with the way pairwise comparisons are used and the way the AHP evaluates alternatives. First, Belton and Gear [13], observed that the AHP may reverse the ranking of the alternatives when an alternative identical to one of the already existing alternatives is introduced. Later, Saaty [10], accepted the variant of the AHP and now it is called the Ideal Mode AHP.

The fact that rank reversal also occurs in the AHP when near copies are considered, has also been studied by Dyer and Wendell [2]. Saaty [9], provided some axioms and guidelines on how close a near copy can be to an original alternative without causing a rank reversal. The first step in the AHP is the estimation of the pertinent data, that is, the estimation of the a_{ij} and W_j values of the decision matrix.

Table 1: Scale of Relative Importance (according to Saaty (1980))

Intensity of Importance	**Definition**	**Explanation**
1	Equal importance	Two activities contribute equally to the objective
3	Weak importance of one over another	Experience and judgment slightly favor one activity over another
5	Essential or strong importance	Experience and judgment strongly favor one activity over another
7	Demonstrated importance	An activity is strongly favored and its dominance demonstrated in practice
9	Absolute importance	The evidence favoring one activity over another is of the highest possible order of affirmation
2,4,6,8	Intermediate values between the two adjacent judgments	When compromise is needed
Reciprocals of above nonzero	If activity i has one of the above nonzero numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i.	

3.1 THE USE OF PAIRWISE COMPARISONS

Pairwise comparisons are used to determine the relative importance of each alternative in terms of each criterion. In this approach the decision-maker has to express his opinion about the value of one single pairwise comparison at a time. Each choice is a linguistic phrase.

Pairwise comparisons are quantified by using a scale. The scale proposed by Saaty is depicted in Table 1. The values of the pairwise comparisons in the AHP are determined according to the scale introduced by Saaty [11].

Suppose that in the example of selecting the best computer system, there are three alternative configurations, say A, B, and C. Also, suppose that one of the decision criteria is hardware expandability (i.e., the flexibility of attaching to the system other related peripheral devices, such as printers, new memory, etc.).

Table 2: Judgment matrix when the three alternative configurations are examined.

C1: Hardware Expandability	A	B	C
A	1	6	8
B	1/6	1	2
C	1/8	1/2	1

The next step is to extract the relative importance implied by the comparisons. That is, how important are the three alternatives when they are considered in terms of the hardware expandability criterion? Saaty asserts that to answer this question one has to estimate the right principal eigenvector of the previous matrix. Hence, for the previous matrix (Table 2) the corresponding priority vector is: (0.769, 0.147, 0.084). An evaluation of the eigenvalue approach can be found in [1]. The consistency ratio (CR) coefficient is calculated as follows. First the consistency index (CI) needs to be estimated that involves approximation of the maximum eigenvalue, denoted by λ_{max}. Then, the CI value is calculated by using the formula: CI = $(\lambda_{max} - n)/ (n - 1)$.

Next the consistency ratio CR is obtained by dividing the CI value by the Random Consistency index (RCI) as given in Table 3.

Table 3: RCI values for different values of n.

n	1	2	3	4	5	6	7	8	9
RCI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45

When these approximations are applied to the judgment matrix it can be verified that the following are derived: λ_{max} = 3.020, CI = 0.010 and CR = 0.017.

After the alternatives are compared with each other in terms of each one of the decision criteria and the individual priority vectors are derived, the synthesis step is taken. The priority vectors become the column of the decision matrix. Finally, given a decision matrix the final priorities, denoted by A^i_{AHP}, of the alternatives in terms of all the criteria combined are determined according to the following formula (1).

$$A^i_{AHP} = \sum_{j=1}^{N} aij \, . \, wj \quad ; \text{for all i=1, 2, 3... M.} \qquad (1)$$

3.2 A NUMERICAL EXAMPLE

Suppose that the three alternative computer systems described earlier need to be evaluated in terms of the four decision criteria: hardware expandability, hardware maintainability, financing available, and user friendly characteristics of the operating system and related available software. If more criteria are required to be considered, then this example can be expanded accordingly. Suppose that the following matrices represent the corresponding judgment matrices with the pairwise comparisons. The corresponding priority vectors (for the individual criteria) and the consistency coefficients are given below.

Table 4: Hardware Expandability Criterion;
λ_{max} = 3.020, CI = 0.010, and CR = 0.017.

C1: Hardware Expandability	A	B	C	**Priority Vector**
A	1	6	8	0.769
B	1/6	1	2	0.147
C	1/8	1/2	1	0.084

Table 5: Hardware Maintainability Criterion;
λ_{max} = 3.551, CI = 0.276, and CR = 0.476.

C2: Hardware Maintainability	A	B	C	**Priority Vector**
A	1	7	1/5	0.233
B	1/7	1	1/8	0.055
C	5	8	1	0.713

Table 6: Financing Available Criterion;
λ_{max} = 3.139, CI = 0.070, and CR = 0.121.

C3: Financing Available	A	B	C	**Priority Vector**
A	1	8	6	0.754
B	1/8	1	1/4	0.065
C	1/6	4	1	0.181

Table 7: User Friendly Criterion; $\lambda_{max} = 3.086$, CI = 0.043, and CR = 0.074.

C4: User Friendly	A	B	C	**Priority Vector**
A	1	5	4	0.674
B	1/5	1	1/3	0.101
C	1/4	3	1	0.226

Finally, the following (Table 8) is the judgment matrix for the case of comparing the importance of the four decision criteria.

Table 8: The Four Criteria; $\lambda_{max} = 4.232$, CI = 0.077, and CR = 0.133.

The Four Criteria	C_1	C_2	C_3	C_4	**Priority Vector**
C_1	1	5	3	7	0.553
C_2	1/5	1	1/3	5	0.131
C_3	1/3	3	1	6	0.270
C_4	1/7	1/5	1/6	1	0.045

The priority vectors are used to form the entries of the decision matrix for this problem. The decision matrix and the resulted final priorities (which are calculated according to formula (1)) are as follows:

Table 9: Decision Matrix and Solution when the Original AHP is used.

Alternative	**Criterion**				**Final Priority**
	C_1	C_2	C_3	C_4	
	0.553	0.131	0.270	0.045	
A_1	0.769	0.233	0.754	0.674	0.690
A_2	0.147	0.055	0.065	0.101	0.111
A_3	0.084	0.713	0.181	0.226	0.199

Table 10: Decision Matrix and Solution when the Ideal Mode AHP is used.

Alternative	**Criterion**				**Final Priority**	**After Normalization**
	C_1	C_2	C_3	C_4		
	0.553	0.131	0.270	0.045		
A_1	1.000	0.327	1.000	1.000	0.911	0.686
A_2	0.191	0.077	0.086	0.150	0.146	0.110
A_3	0.109	1.000	0.240	0.335	0.271	0.204

Therefore, the best system is A followed by system C which is followed by system B. It must be observed that although both the original AHP and the ideal mode AHP yielded the same ranking for the three alternatives, they assigned different final priorities for these alternatives.

4. TESTING THE AHP OVER EVALUATIVE CRITERIA

Since the best alternative can be same regardless of the various methods chosen, an estimation of the accuracy of each method is highly desirable. The most difficult problem that may arise is how one can evaluate a multi-dimensional decision-making method when the true best alternative is not known. Two evaluative criteria were introduced for the above purpose.

The first evaluative criterion has to do with the premise that a method which is accurate in multi-dimensional problems should also be accurate in single-dimensional problems.

The second evaluative criterion considers the premise that a desirable method should not change the indication of the best alternative when an alternative (not the best) is replaced by another worse alternative (given that the importance of each criterion remains unchanged).

4.1 TESTING THE ORIGINAL AHP USING THE FIRST EVALUATIVE CRITERION

Example 1: Suppose that the matrix below depicts the actual values of three alternatives A1, A2, and A3, in terms of three criteria with the following weights of importance: w1 = 8/13, w2 = 1/13, and w3 = 3/13.

Table 11: Matrix showing the values of alternatives.

Alternative	**Criterion**		
	C_1	C_2	C_3
	8/13	1/13	3/13
A_1	1	9	9
A_2	5	2	2
A_3	1	5	9

Given the above data it is easy to see that the final scores of the alternatives in terms of the three criteria are 44/13, 48/13, and 40/13, respectively. For instance, the score for the first alternative is: 1(8/13) + 9(1/13) + 9(3/13) = 44/13. Therefore, alternative A_2 is the best one (since it corresponds to the highest score 48/13). Since there are three criteria, the decision maker needs to construct three matrices with pairwise comparisons of size 3x3 each. The three 3x3 matrices with the pairwise comparisons that correspond to this problem are as follows:

Criterion C1			Criterion C2			Criterion C3		
1	1/5	1/1	1	9/2	9/5	1	9/2	9/9
5	1	5/1	2/9	1	2/5	2/9	1	2/9
1	1/5	1	5/9	5/2	1	9/9	9/2	1

Table 12: Relative importance matrix of the alternatives

Alternative	**Criterion**		
	C_1	C_2	C_3
	8/13	1/13	3/13
A_1	1/7	9/16	9/20
A_2	5/7	2/16	2/20
A_3	1/7	5/16	9/20

Applying the last step of the AHP it turns out that the alternative A_2 is the best one ($A^2_{AHP} = A^*_{AHP} = 0.472$). This verifies the AHP test over the first evaluative criterion.

4.2 TESTING THE ORIGINAL AHP USING THE SECOND EVALUATIVE CRITERION

Example 2: Suppose that the following is a matrix that contains relative values for the importance of the alternatives. Assume that the criteria have weights w1 = 2/7, w2 = 1/7, and w3 = 3/7.

Table 13: Relative importance matrix of the alternatives M1:

Alternative	**Criterion**		
	C_1	C_2	C_3
	2/7	1/7	3/7
A_1	9/19	2/12	2/7
A_2	5/19	1/12	4/7
A_3	5/19	9/12	1/7

The priority vectors of the alternative for matrix M1 are (0.282, 0.332, 0.244). Apparently, the best alternative is A_2. If in the above problem the alternative A_1 (which is not the best one and was defined by the relative values 9/19, 2/12, 2/7), is replaced by A'_1 which is worse than the original alternative A_1, then, the above matrix is modified as follows:

Table 14: Relative importance matrix of the alternatives M2:

Alternative	**Criterion**		
	C_1	C_2	C_3
	2/7	1/7	3/7
A'_1	8/18	1/11	1/6
A_2	5/18	1/11	4/6
A_3	5/18	9/11	1/6

Matrix M2 has been derived from matrix M1 by substituting the alternative A_1 with the lesser A'_1 = (8/18 1/11 1/6) < (9/19 2/12 2/7). Similarly, the priority vector for matrix M2 is (0.211, 0.378, 0.268). It is clear that now the best alternative is A_2.Thus, the introduction of a new worse alternative (different from the best one) does not change the indication of the best alternative. This verifies the AHP test over the second evaluative criterion.

4 CONCLUSION

The AHP is a decision support tool which can be used to solve complex decision problems. It provides a convenient approach for solving MCDM problems in engineering. There is sufficient evidence to suggest that the recommendations made by the AHP should not be taken literally. In matter of fact, the closer the final priority values are with each other, the more careful the user should be. This is true with any MCDM method. The numerical examples in this paper strongly suggest that when some alternatives appear to be very close with each other, then the decision-maker needs to be very cautious. An apparent remedy is to try to consider additional decision criteria which, hopefully, can assist in drastically discriminating among the alternatives. The above observations suggest that MCDM methods should be used as decision support tools and as the means for deriving the final answer. The research in this area of decision-making is still very valuable in many scientific and engineering applications.

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